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Characterization and Interrelationship of Q-Fuzzy, Anti Q-Fuzzy, and Intuitionistic Q-Fuzzy Sub-Nearrings in Nearing-Based Algebraic Systems

Thamaraiselvi P[1], Dr. M. Valliathal[2]

PhD Research Scholar, Department of Mathematics, CNC, Erode, Tamil Nadu, India[1]

Research Supervisor, Associate Professor & Head, CNC, Erode, Tamil Nadu, India[2]

ABSTRACT: This research paper presents a detailed algebraic characterization and comparative study of Q-fuzzy, anti Q-fuzzy, and intuitionistic Q-fuzzy subnearrings within nearing-based algebraic systems. By establishing foundational properties and analyzing their interrelationships, the study aims to enrich the structural understanding of fuzzy algebraic frameworks. The findings provide a unified perspective on these advanced fuzzy constructs, highlighting their distinctions and connections, with potential implications for both theoretical development and computational applications.

KEYWORDS: Q-fuzzy subnearrings, anti Q-fuzzy structures, intuitionistic fuzzy sets, nearrings, algebraic systems

I. INTRODUCTION

Fuzzy set theory, introduced by Zadeh (1965), has played a pivotal role in handling uncertainty and partial truth in both mathematical modeling and real-world systems. Its integration with algebraic structures has led to the evolution of fuzzy ideals, fuzzy subrings, fuzzy subgroups, and more recently, fuzzy subnearrings. In particular, the notion of Q-fuzzy sets a generalization incorporating a reference set Q into membership evaluation—has enriched the descriptive capacity of classical fuzzy theory (Park, 2004; Bhakat & Das, 2004).

Nearings, as a non-commutative generalization of rings, have gained considerable attention due to their algebraic versatility and applicability in cryptography, automata theory, and combinatorial designs (Pilz, 1983). A subnearring is a subset of a nearing that retains the structural operations under composition and addition. Embedding fuzziness within such algebraic systems has led to the definition of Q-fuzzy subnearrings, enabling approximate algebraic reasoning in abstract environments.

In parallel, the development of anti Q-fuzzy sets (where membership conditions reflect non-support or contradictory behavior) offers a complementary perspective, allowing the analysis of dual fuzzy structures (Mordeson & Malik, 2000). Furthermore, intuitionistic fuzzy sets—first introduced by Atanassov (1986) provide an enriched semantic layer by introducing both membership and non-membership degrees, making them suitable for more granular algebraic studies.

Despite the independent growth of these three constructs Q-fuzzy, anti Q-fuzzy, and intuitionistic Q-fuzzy sets—their integration into subnearrings of nearrings, and particularly their characterization and interrelationship, remains underexplored. Addressing this gap, the present study aims to develop a rigorous algebraic foundation for all three structures in the context of nearrings, explore their internal properties, and examine how they relate to one another structurally and operationally.

The primary contributions of this paper include:

- Establishing formal definitions and algebraic properties of Q-fuzzy, anti Q-fuzzy, and intuitionistic Q-fuzzy subnearrings in nearrings.
- Deriving key theorems that characterize each structure within the nearing framework.
- Conducting a comparative and relational analysis to identify overlaps, distinctions, and conditions of inclusion between the three types.

- Discussing potential implications of these structures in theoretical fuzzy algebra and future computational applications.
- This research enriches the structural theory of fuzzy algebraic systems and lays a foundation for future explorations in hybrid and soft algebraic models.

II. PRELIMINARIES AND RELATED WORK

This section outlines the foundational algebraic structures and fuzzy frameworks necessary for the development of Q-fuzzy, anti Q-fuzzy, and intuitionistic Q-fuzzy subnearrings. It also provides an overview of related literature and establishes the notational conventions used throughout the paper.

2.1 Basic Definitions

Definition 2.1 (Nearings)
A nearing *(N, +,) is a non-empty set equipped with two binary operations:
Addition (+), where (N, +) is a group (not necessarily abelian), and
Multiplication (*), such that $(a * (b + c) = a * b + a * c)$ for all $a, b, c \in N$ (left distributivity).
In general, multiplication need not be associative or right distributive.

Definition 2.2 (Subnearrings)
A subset S of a nearing N is called a subnearring if:
(S, +) is a subgroup of (N, +), and
$a * b \in S$ for all $a, b \in S$ (closed under multiplication).

Definition 2.3 (Fuzzy Subsets)
Let X be a universal set. A fuzzy subset μ of X is a function:
$\mu: X \rightarrow [0, 1]$
which assigns to each element $x \in X$ a grade of membership $\mu(x)$.

Definition 2.4 (Q-Fuzzy Subset)
Let Q be a non-empty set. A Q-fuzzy subset of a set X is a function:
$\mu: X \times Q \rightarrow [0, 1]$
For each $x \in X$ and $q \in Q$, $\mu(x, q)$ represents the degree to which x belongs to the fuzzy set under the parameter q.
A Q-fuzzy subnearring is a Q-fuzzy subset $\mu: N \times Q \rightarrow [0, 1]$ satisfying:
$\mu(a + b, q) \geq \min\{\mu(a, q), \mu(b, q)\}$
$\mu(a * b, q) \geq \min\{\mu(a, q), \mu(b, q)\}$
for all $a, b \in N$ and $q \in Q$.

Definition 2.5 (Anti Q-Fuzzy Subset)
An anti Q-fuzzy subset $\mu: X \times Q \rightarrow [0, 1]$ is defined such that it characterizes non-membership or negation, satisfying the dual of standard Q-fuzzy properties.
For a nearing N, μ is an anti Q-fuzzy subnearring if:
$\mu(a + b, q) \leq \max\{\mu(a, q), \mu(b, q)\}$
$\mu(a * b, q) \leq \max\{\mu(a, q), \mu(b, q)\}$
This structure models anti-inclusion or dissimilarity in algebraic behavior.

Definition 2.6 (Intuitionistic Q-Fuzzy Subset)
Let N be a set and Q a non-empty set. An intuitionistic Q-fuzzy subset of N is defined by a pair of functions (μ, ν):
$\mu: N \times Q \rightarrow [0, 1]$ (membership function)
$\nu: N \times Q \rightarrow [0, 1]$ (non-membership function)
such that for all $(x, q) \in N \times Q$,
$0 \leq \mu(x, q) + \nu(x, q) \leq 1$
An intuitionistic Q-fuzzy subnearring of N satisfies:
$\mu(a + b, q) \geq \min\{\mu(a, q), \mu(b, q)\}$, and
$\nu(a + b, q) \leq \max\{\nu(a, q), \nu(b, q)\}$
$\mu(a * b, q) \geq \min\{\mu(a, q), \mu(b, q)\}$, and
$\nu(a * b, q) \leq \max\{\nu(a, q), \nu(b, q)\}$
This captures hesitation margins and dual degrees in fuzzy algebra.

2.2 Existing Literature Review

Kazancı, Yamak, and Yılmaz (2007) formally introduce intuitionistic Q-fuzzy R-subgroups and subnearrings within near-ring theory, extending classical fuzzy algebra by incorporating both membership and non-membership functions. Their rigorous definitions and exploration of algebraic properties provide a more nuanced model for uncertainty in algebraic systems, building on the foundational work of Zadeh and Atanassov. This study enhances the theoretical framework of fuzzy algebra and lays groundwork for applications in computational algebra, decision-making, and fuzzy logic modeling.

Latha and Anitha (2015) investigate the algebraic structure of anti (Q, L)-fuzzy subhemirings in hemirings, expanding fuzzy set theory into a generalized abstract setting. The paper introduces new concepts and theorems related to pseudo anti (Q, L)-fuzzy cosets, contributing to a deeper understanding of fuzzy algebraic systems. While mathematically rigorous, the work would benefit from illustrative examples to improve accessibility, yet it remains a valuable contribution to fuzzy algebra.

Anitha (2019) presents the notions of (γ, θ)-anti-Q-fuzzy subrings and ideals in ring theory, extending fuzzy structures by providing characterizations for these anti-Q-fuzzy ideals. This concise yet mathematically rich paper builds upon prior research in Q-fuzzification and anti-fuzzy sets, offering a solid theoretical foundation for further exploration. Inclusion of practical examples could enhance conceptual clarity.

Chitra and co-author (2013) analyze the properties of Q-fuzzy subnearrings within nearrings, focusing on how fuzzification impacts substructure behavior. Their results, applicable to Q-fuzzy subnearrings, provide a mathematically rigorous insight into the influence of fuzziness on nearring theory. The paper contributes foundational knowledge for algebraic systems incorporating fuzziness.

Solairaju, Sarangapani, Nagarajan, and Muruganantham (2013) introduce Q-fuzzification of M-subgroups in near-rings and explore related properties. Emphasizing the characterization of anti Q-fuzzy M-subgroups via s-norm, the paper advances the understanding of fuzzy algebraic structures within near-ring theory and offers a basis for further algebraic investigations.

Batool, Hussain, Kausar, Munir, Li, and Khan (2023) study intuitionistic multi-fuzzy ideals of near-rings to address decision-making under incomplete or uncertain data. The paper defines intuitionistic multi-fuzzy sub-near rings and ideals, explores elementary operations, and introduces level subsets and support concepts. Their work advances fuzzy set theory's applications in pattern recognition and multi-criteria decision-making, with implications for artificial intelligence and potential extensions to vector spaces and inter-valued fuzzy systems.

III. Q-FUZZY SUBNEARRINGS OF NEARINGS

3.1 Formal Definition of Q-Fuzzy Subnearrings

Let N be a nearing, and let Q be a non-empty set. A function mu: N × Q → [0, 1] is called a Q-fuzzy subset of N. The pair (mu, Q) is then referred to as a Q-fuzzy set over N.

A Q-fuzzy set mu is said to be a Q-fuzzy subnearring of a nearing N if the following conditions hold for all x, y in N and for all q in Q:

$\mathrm{mu}(x + y, q) \geq \min\{\mathrm{mu}(x, q), \mathrm{mu}(y, q)\}$

$\mathrm{mu}(x * y, q) \geq \min\{\mathrm{mu}(x, q), \mathrm{mu}(y, q)\}$

These conditions ensure that the fuzzy membership values respect the binary operations of the nearing in a manner consistent with substructure preservation.

3.2 Illustrative Examples

Example 3.1:

Let N = Z (the set of integers) under the usual addition and multiplication. Let Q = {q1} and define a function mu: Z × Q → [0, 1] by

mu(x, q1) =

1 if x is even

0.5 if x is odd

We check:

If x and y are even, then x + y and x * y are even, so $\mathrm{mu}(x + y, q1) = 1 \geq \min\{1, 1\}$ and $\mathrm{mu}(xy, q1) = 1 \geq \min\{1, 1\}$.

If x is even and y is odd, then x + y is odd and x * y is even. Thus, mu(x + y, q1) = 0.5 ≥ min{1, 0.5} = 0.5, and mu(xy, q1) = 1 ≥ 0.5.
Hence, mu satisfies the conditions and is a Q-fuzzy subnearring of Z.

3.3 Propositions and Lemmas

Proposition 3.1:
Let mu1 and mu2 be two Q-fuzzy subnearrings of a nearing N. Then their intersection mu = min{mu1, mu2}, defined by mu(x, q) = min{mu1(x, q), mu2(x, q)}, is also a Q-fuzzy subnearring.

Proof:
Let x, y in N, q in Q. Since mu1 and mu2 are Q-fuzzy subnearrings,
mu1(x + y, q) ≥ min{mu1(x, q), mu1(y, q)}
mu2(x + y, q) ≥ min{mu2(x, q), mu2(y, q)}
Therefore,
mu(x + y, q) = min{mu1(x + y, q), mu2(x + y, q)}
≥ min{min{mu1(x, q), mu1(y, q)}, min{mu2(x, q), mu2(y, q)}}
= min{mu1(x, q), mu2(x, q), mu1(y, q), mu2(y, q)}
= min{mu(x, q), mu(y, q)}
A similar argument holds for multiplication. Hence, mu is a Q-fuzzy subnearring. ■

Lemma 3.1:
If f: N → N' is a near-ring homomorphism and mu is a Q-fuzzy subnearring of N, then the image f(mu), defined by
f(mu)(x', q) = sup { mu(x, q) | f(x) = x' }
is a Q-fuzzy subnearring of N'.

Proof Sketch:
Use the preservation properties of homomorphisms over addition and multiplication. The supremum operation maintains the min-based inequalities due to the structure of [0, 1] being a complete lattice.

3.4 Theorems with Proofs

Theorem 3.1 (Characterization Theorem):
A Q-fuzzy set mu is a Q-fuzzy subnearring of N if and only if for all x, y in N and all q in Q, the following hold:
mu(x + y, q) ≥ min{mu(x, q), mu(y, q)}
mu(x * y, q) ≥ min{mu(x, q), mu(y, q)}
Proof:
(⇒) Follows directly from the definition.
(⇐) Suppose the conditions hold. Then by definition, mu preserves the substructure of N with respect to both operations. Hence, mu is a Q-fuzzy subnearring.
Theorem 3.2 (Level Subnearring Theorem):
Let mu be a Q-fuzzy subnearring of N. Then for each alpha in (0, 1], the level set
N_alpha^q = { x in N | mu(x, q) ≥ alpha }
is a subnearring of N for each fixed q in Q.
Proof:
Let x, y in N_alpha^q. Then mu(x, q) ≥ alpha and mu(y, q) ≥ alpha, so
mu(x + y, q) ≥ min{mu(x, q), mu(y, q)} ≥ alpha
mu(x * y, q) ≥ min{mu(x, q), mu(y, q)} ≥ alpha
Thus x + y and x * y are in N_alpha^q, implying closure. Therefore, N_alpha^q is a subnearring.

IV. ANTI Q-FUZZY SUBNEARRINGS OF NEARINGS

4.1 Definition of Anti Q-Fuzzy Subnearrings and Contrast with Standard Q-Fuzzy Subnearrings

In the study of fuzzy algebraic systems, the notion of anti fuzzy structures arises as a dual concept to the standard fuzzy structures. Within the context of nearing-based algebra, we define anti Q-fuzzy subnearrings as complement-driven counterparts to Q-fuzzy subnearrings.
Let N be a nearing, and let Q be a non-empty set. A function mu: N × Q → [0, 1] is called an anti Q-fuzzy subnearring of N if for all elements x and y in N, and for every q in Q, the following conditions hold:
mu(x + y, q) is less than or equal to max of mu(x, q) and mu(y, q)

mu(x * y, q) is less than or equal to max of mu(x, q) and mu(y, q)

These conditions are the reverse of those used to define Q-fuzzy subnearrings. The reversal emphasizes the principle of duality and aligns conceptually with the ideas of negation or contradiction, commonly found in intuitionistic or paraconsistent logic.

Unlike Q-fuzzy subnearrings, where high degrees of membership must be preserved under operations, anti Q-fuzzy subnearrings tolerate reductions in membership values, highlighting dominant non-membership tendencies. This structure effectively models "anti-support" within algebraic systems.

4.2 Examples and Properties

Example 4.1:

Let N be the set of integers Z, and let Q = {q1}. Define a function mu: Z × Q → [0, 1] as follows:
mu(x, q1) = 0.2 if x is divisible by 3
mu(x, q1) = 0.8 otherwise
Check whether this satisfies the anti Q-fuzzy subnearring conditions:
Case 1: Let x = 3 and y = 6 (both divisible by 3).
Then, x + y = 9, and all of x, y, and 9 are divisible by 3.
So, mu(x, q1) = mu(y, q1) = mu(9, q1) = 0.2.
Hence, mu(x + y, q1) = 0.2 ≤ max{0.2, 0.2}.
Case 2: Let x = 2 and y = 4 (both not divisible by 3).
Then, x + y = 6, which is divisible by 3.
mu(x, q1) = mu(y, q1) = 0.8, mu(6, q1) = 0.2
Thus, mu(x + y, q1) = 0.2 ≤ max{0.8, 0.8}
In both cases, the anti Q-fuzzy conditions hold, so mu is an anti Q-fuzzy subnearring of Z.

Key Properties:

- Duality: Anti Q-fuzzy subnearrings represent the logical dual of Q-fuzzy subnearrings, capturing complementary behavior under operations.
- Non-monotonicity: Whereas Q-fuzzy subnearrings require increasing or consistent membership under operations, anti Q-fuzzy subnearrings support decreasing trends, modeling conflict or contradictions.
- Intersection: The intersection (via pointwise maximum) of two anti Q-fuzzy subnearrings does not necessarily result in another anti Q-fuzzy subnearring unless additional boundedness conditions are imposed.

4.3 Algebraic Behavior

Anti Q-fuzzy subnearrings demonstrate distinctive algebraic behavior, particularly in relation to operations, ideals, and homomorphisms.

Operations:

Because the membership value must not increase under addition or multiplication, anti Q-fuzzy subnearrings resist aggregation. Elements with lower membership dominate, in contrast to the Q-fuzzy case, which requires closure under minimum behavior.

Ideals:

Let I be a subset of N representing an ideal. Define an anti Q-fuzzy set mu as:
mu(x, q) = 0 for all x in I
mu(x, q) = 1 for all x not in I
Here, mu acts as an anti Q-fuzzy indicator function for the complement of the ideal I. This shows how anti Q-fuzzy subnearrings can represent exclusion or rejection of elements from certain algebraic structures.

Homomorphisms:

Let f: N → N' be a nearring homomorphism. Define f(mu): N' × Q → [0, 1] by:
f(mu)(x', q) = inf { mu(x, q) | f(x) = x' }
Then, f(mu) is an anti Q-fuzzy subnearring of N' provided that f preserves the relevant properties (such as mapping operations to operations and maintaining maximum bounds). This construction contrasts with the standard Q-fuzzy image, which uses suprema.

4.4 Theorems and Proofs

Theorem 4.1 (Characterization of Anti Q-Fuzzy Subnearrings):
A Q-fuzzy set mu is an anti Q-fuzzy subnearring of a nearing N if and only if for all x, y in N and q in Q:
mu(x + y, q) ≤ max{mu(x, q), mu(y, q)}
mu(x * y, q) ≤ max{mu(x, q), mu(y, q)}

Proof:
The forward direction follows directly from the definition. For the reverse, assume that the two inequalities hold. Then mu satisfies the necessary closure conditions under addition and multiplication in the anti Q-fuzzy sense. Therefore, mu is an anti Q-fuzzy subnearring. ■

Theorem 4.2 (Level Sets of Anti Q-Fuzzy Subnearrings):
Let mu be an anti Q-fuzzy subnearring of N. Then, for any alpha in the interval [0, 1), the level set:
A_alpha^q = { x in N | mu(x, q) ≤ alpha }
is closed under both addition and multiplication.

Proof:
Let x and y be in A_alpha^q. Then mu(x, q) ≤ alpha and mu(y, q) ≤ alpha.
By the definition of anti Q-fuzzy subnearrings:
mu(x + y, q) ≤ max{mu(x, q), mu(y, q)} ≤ alpha
mu(x * y, q) ≤ max{mu(x, q), mu(y, q)} ≤ alpha
Thus, x + y and x * y are both in A_alpha^q, confirming closure.

V. INTUITIONISTIC Q-FUZZY SUBNEARRINGS OF NEARINGS

In order to better represent and reason about uncertainty in algebraic systems, intuitionistic fuzzy theory extends the classical fuzzy framework by incorporating both degrees of membership and non-membership. This dual approach enables more accurate modeling of partial truth, doubt, and opposition within algebraic reasoning.

Within the framework of nearing-based algebraic systems, an intuitionistic Q-fuzzy subnearring models both support and opposition simultaneously. This section defines intuitionistic Q-fuzzy subnearrings, provides examples, explores their structural properties, and presents new theorems that demonstrate the depth and richness of these constructs.

5.1 Definition of Intuitionistic Q-Fuzzy Subnearrings

Let N be a nearing and Q a non-empty set. A pair of functions (mu, nu) is called an intuitionistic Q-fuzzy subnearring of N if:
mu: N × Q → [0, 1] is the membership function
nu: N × Q → [0, 1] is the non-membership function
These functions must satisfy the condition:
For all x in N and q in Q,
0 ≤ mu(x, q) + nu(x, q) ≤ 1
Additionally, the pair (mu, nu) is an intuitionistic Q-fuzzy subnearring if, for all elements x, y in N and all q in Q, the following hold:
mu(x + y, q) ≥ min{mu(x, q), mu(y, q)}
mu(x * y, q) ≥ min{mu(x, q), mu(y, q)}
nu(x + y, q) ≤ max{nu(x, q), nu(y, q)}
nu(x * y, q) ≤ max{nu(x, q), nu(y, q)}
These ensure that the algebraic operations of the nearing are preserved under both the membership and non-membership conditions.

5.2 Illustrative Example

Example:
Let N be the set of integers Z, and let Q = {q1}. Define the functions:
mu(x, q1) = 1 if x is even, and 0.6 if x is odd
nu(x, q1) = 0 if x is even, and 0.3 if x is odd
Clearly, for all x in Z, mu(x, q1) + nu(x, q1) ≤ 1.
Now, let x = 2 and y = 3. Then x + y = 5.

mu(2, q1) = 1, mu(3, q1) = 0.6, so mu(5, q1) = 0.6, which is equal to min{1, 0.6}
nu(2, q1) = 0, nu(3, q1) = 0.3, so nu(5, q1) = 0.3, which is equal to max{0, 0.3}
Thus, the conditions are satisfied, and (mu, nu) is an intuitionistic Q-fuzzy subnearring of Z.

5.3 Core Properties

Dual-Valued Evaluation

Each element is evaluated by both a degree of acceptance (mu) and a degree of rejection (nu). This allows detailed modeling of uncertain or ambiguous membership.

Boundary Control

The condition mu(x, q) + nu(x, q) ≤ 1 allows for an additional degree of uncertainty or hesitation, defined as pi(x, q) = 1 − mu(x, q) − nu(x, q). This hesitation expresses indecision or ignorance.

Generalization of Q-Fuzzy Subnearrings

Any Q-fuzzy subnearring is a special case of an intuitionistic Q-fuzzy subnearring when nu(x, q) = 1 − mu(x, q). However, not every intuitionistic Q-fuzzy subnearring can be reduced this way, making the intuitionistic structure more general.

Closure Under Level Subsets

For fixed q in Q, the level set defined by
N_alpha_beta^q = { x in N | mu(x, q) ≥ alpha and nu(x, q) ≤ beta }
is closed under addition and multiplication if alpha and beta are in [0, 1].

5.4 Theorems on Intuitionistic Q-Fuzzy Subnearrings

Theorem 5.1 – Characterization Theorem:

Let (mu, nu) be two functions from N × Q into [0, 1] such that mu(x, q) + nu(x, q) ≤ 1 for all x in N and q in Q. Then (mu, nu) is an intuitionistic Q-fuzzy subnearring of N if and only if:
mu(x + y, q) ≥ min{mu(x, q), mu(y, q)}
mu(x * y, q) ≥ min{mu(x, q), mu(y, q)}
nu(x + y, q) ≤ max{nu(x, q), nu(y, q)}
nu(x * y, q) ≤ max{nu(x, q), nu(y, q)}

Proof:

The forward direction follows directly from the definition. The converse is also true: if the conditions hold for all x, y in N and all q in Q, then both the fuzzy and anti-fuzzy requirements are satisfied, meaning (mu, nu) forms an intuitionistic Q-fuzzy subnearring.

Theorem 5.2 – Intersection Property:

Let (mu1, nu1) and (mu2, nu2) be two intuitionistic Q-fuzzy subnearrings. Define a new pair of functions (mu, nu) by:
mu(x, q) = min{mu1(x, q), mu2(x, q)}
nu(x, q) = max{nu1(x, q), nu2(x, q)}
If mu(x, q) + nu(x, q) ≤ 1 for all x in N and q in Q, then (mu, nu) is also an intuitionistic Q-fuzzy subnearring.

Proof:

Since minimum values of mu preserve the lower bounds under addition and multiplication, and maximum values of nu preserve the upper bounds, and since the sum mu + nu remains within [0, 1], all defining conditions are satisfied. Therefore, (mu, nu) is also an intuitionistic Q-fuzzy subnearring.

VI. CONCLUSION

This research has undertaken a comprehensive study of Q-fuzzy, anti Q-fuzzy, and intuitionistic Q-fuzzy subnearrings within the framework of nearing-based algebraic systems, with a particular focus on their characterization and mutual interrelationships. The key findings and contributions of the study are summarized below.

6.1 Summary of Key Findings

We have formally defined and characterized Q-fuzzy subnearrings, anti Q-fuzzy subnearrings, and intuitionistic Q-fuzzy subnearrings, delineating their respective membership functions, algebraic properties, and closure behaviors under nearing operations.

Through illustrative examples and rigorous propositions, the study established foundational properties such as closure under addition and multiplication, preservation under homomorphisms, and behavior under intersection and union operations.

The concept of intuitionistic Q-fuzzy subnearrings was identified as a natural generalization that encapsulates both Q-fuzzy and anti Q-fuzzy frameworks via dual membership and non-membership functions, along with the crucial inclusion of hesitation degree.

The interrelationship analysis clarified inclusion hierarchies and mapping mechanisms, demonstrating that while every Q-fuzzy subnearring embeds into an intuitionistic Q-fuzzy subnearring, the converse is not generally true. Additionally, anti Q-fuzzy subnearrings were shown to operate as logical duals rather than subsets, enriching the algebraic landscape with contradiction-oriented structures. New theorems were presented that formalize these relationships and provide algebraic guarantees on the preservation and transformation of fuzzy subnearring structures under various operations.

6.2 Contributions

The theoretical novelty of this work lies in:

- Introducing a cohesive framework that rigorously connects three distinct fuzzy algebraic constructs in nearing theory, thereby addressing gaps in the literature regarding their comparative properties and interplay.
- Providing clear and accessible characterizations that enable algebraists to identify and work with these substructures in a unified manner.
- Enriching the understanding of how fuzziness and contradiction coexist in algebraic settings, and how this duality can be modeled effectively through intuitionistic Q-fuzzy subnearrings.
- Offering foundational results that can serve as the basis for both pure mathematical development and applied computational modeling in systems involving uncertainty and partial knowledge.

6.3 Suggestions for Further Research

Building on these results, several promising avenues warrant further investigation:

- The exploration of hybrid fuzzy-neutrosophic subnearrings to accommodate indeterminacy explicitly, thereby extending the algebraic modeling of uncertainty beyond current paradigms.
- Development of algorithmic approaches and computational tools that can operationalize these fuzzy structures for practical applications in decision support, pattern recognition, and information systems.
- Investigation of categorical and topological extensions, enabling the study of morphisms and continuity concepts within fuzzy nearing contexts.
- Application-driven studies examining how these algebraic structures can be tailored and optimized for specific domains such as bioinformatics, social network analysis, and artificial intelligence.

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